大 气 科 学 1998
Chinese Journal of Atmospheric Sciences
Volume 22, No.4, pp. 348–360

The Qualitative Theory on the Dynamical Equations of Atmospheric Motion and Its Applications

Li Jianping(李建平)
(State Key Laboratory of Numerical Modeling for Atmospheric Sciences and Geophysical Fluid Dynamics, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing 100029)
Chou Jifan(丑纪范)
(Department of Atmospheric Sciences, Lanzhou University, Lanzhou 730000)

Chinese manuscript received Novermber 27, 1997; revised February, 1998.

Based on the complete dynamical equations of the moist atmospheric motion, the qualitative theory of nonlinear atmosphere with dissipation and external forcing and its applications are discussed systematically by new theories and methods on the infinite dimensional dynamical system. The completely nonlinear dynamical equations of the moist atmospheric motion with dissipation and external forcing are rewritten as an equivalent operator equation (OEq) in Hilbert space, and the properties of operators and their physical senses are studied. On the basis of them, the existence of global attractor (GA) of the moist atmospheric system is proved, and the characteristic of nonlinear adjustment to external forcing is revealed. Furthermore, the results mentioned above are extended to the cases with topographic dynamical effect and non–stationary external forcing. Meanwhile, the existence of the inertial manifold of atmospheric equations is discussed. On the basis of theoretical results, this paper presents three classes of time boundary layers (TBLs) existing in the forced dissipative nonlinear (FDN) system, the principle of simplification of atmospheric equations, the constraint principle of operator for splitting algorithm and the constructive method of a few freedoms to support the base of attractor. Besides, the applications of the theoretical results to the design of difference scheme and the computational stability (CS) analysis of nonlinear evolutionary equations and the numerical analysis of atmospheric multiple equilibria (MEs) and the improvement of extended forecast of numerical models and the short–term climate forecast and the analysis and prediction of one class of mesoscale system are discussed, and the necessary conditions possessed in the dynamic models describing the long–time process are pointed out, and a rational explanation of the harmonization of initial values with model is given. Finally, the future studies are prospected.

Key words: Dynamical equations of moist atmospheric motion; forced dissipative nonlinear (FDN) dynamics; global attractor (GA); qualitative analysis; operator; time boundary layer (TBL), multiple equilibria (MEs); difference scheme.

1. INTRODUCTION

The atmosphere is an infinitely dimensional dynamical system. Its evolution can be

© 1999 by Allerton Press, Inc. Authorization to photocopy individual items for internal or personal use, or the internal or personal use of specific clients, is granted by Allerton Press, Inc. for libraries and other users registered with the Copyright Clearance Center (CCC) Transactional Reporting Service, provided that the base fee of $50.00 per copy is paid directly to CCC, 222 Rosewood Drive, Danvers, MA 01923.

described by a set of partial differential equations, and the studies of atmospheric dynamics and numerical weather forecast are just involving these equations and their various simplified forms[1-4]. However, they are very complicated nonlinear equations with dissipation and external forcing. For a long time, there are short of theories and methods used to deal with this kind of system in mathematics, so the traditional dynamical meteorology is limited in the frames of theory of the linear system and the nonlinear conservative system. Unfortunately there are essential drawbacks which are unable to be overcomed in these theories for the long-term process and the climatic variation and the meso- and micro-scale convective systems. Thus the modern atmospheric dynamics calls for new basic theory of dynamics, the forced dissipative nonlinear (FDN) dynamics[1-3,5-8]. It is only the last ten or more years and ago that the investigation of the FDN dynamics began[1,3,5-15,18-24].

The FDN dynamics involves the long-time behavior of system, i.e., the global asymptotic characteristics of solutions of system in the mathematical sense. In the understanding of the global asymptotic characteristics of solutions for nonlinear equations, there are difficulties unable to be overcomed in all of the traditional indirect methods (i.e., the analytic methods), the numerical methods and the experimental methods[3,5-15,18]. Therefore, we have to find the other way, the qualitative theory of differential equations. The properties of solutions and the main characteristics of nonlinear system can be understood directly from the characteristics of equations themselves by the qualitative methods, which do not need to solve analytic expressions of solutions. It seems, thus, that the qualitative studies are not only very necessary but also inevitable for the discussions of the global characteristics of the dynamical equations of atmospheric and oceanic motion.

Chou[1] initiated the qualitative theory on the dynamical equations of FDN atmospheric motion with dissipation and external forcing. He proved the large-scale atmospheric equations can be succinctly written as an operator equation (OEq) in Hilbert space, and studied the properties of operators; and proved that there exists an absorbing set for the large-scale atmospheric system in R^n, no matter what the initial value is the state of system is bound to evolve into it, and that the volume of the final state set is zero[2,3]. Later, the above results were extended respectively to the cases of the infinite dimensional Hilbert space, the large-scale dynamical equations of oceanic motion, the large-scale air-ocean coupled system and the non-stationary external forcing by Refs. [13], [16], [17], and [19]. The unique existence of solutions of the initial problem and the global attractor were obtained, and it is proved that Hausdorff dimension is limited, and its estimated value was given. And then a series of studies were carried out[20-25]. The qualitative theory of dynamical equations of atmospheric and oceanic motion was discussed systematically and completely in the reference [18], and the characteristics of the long-time evolution of system were revealed. On the basis of theoretical results, Chou and his cooperators[11,12,18,26-31] inquired into some concrete applications, and obtained some good results that show broad applied prospect of the qualitative theory of the FDN dynamics.

Much of known work on the qualitative theory was devoted to the dry atmosphere, little is studied at present about the moist atmosphere. The complete atmosphere should be the moist atmosphere because the latent heat of phase change of moisture is the main form of diabatic heating. Based on the complete dynamical equations of the moist atmospheric motion, the qualitative theory of nonlinear atmosphere with dissipation and external forcing and its applications are discussed systematically. This work is both a summary for known studies during the last decade and more and a deepening study. Meanwhile this paper presents some new views and applications.

2. THEORY

2.1 *A Special OEq in Hilbert Space*

Adopting the system of spherical coordinate λ, θ, r(λ, θ, r are the longitude, the latitude and the geocentric distance respectively) and introducing the vector function φ, the completely dynamical equations of the moist atmospheric motion can be written as equivalent OEq as follows[18,22]:

$$\begin{cases} \dfrac{\partial \varphi}{\partial t} + [N(\varphi) + L(\varphi)]\varphi = \xi(\varphi), & (1) \\ \varphi|_{t=0} = \varphi_0. & (2) \end{cases}$$

Because of space limitation, see references [18,22] for the concrete form of every term in Eq.(1). The domain of solutions of the operator equation is the whole atmosphere enveloping the earth. In Hilbert space, $N(\varphi)$ and $L(\varphi)$ have the properties as follows: (1) $N(\varphi)$ is an anti–adjoint operator, $L(\varphi)$ a self–adjoint operator; (2) $L(\varphi)$ is symmetric, $N(\varphi)$ anti–symmetric; (3) $L(\varphi)$ is a positive operator.

The operator $N(\varphi)$ contains the actions of the nonlinear advection, the Coriolis force, the spherical curvature, the pressure–gradient force and the gravity etc. In physical sense, it represents the reversible adiabatic processes of energy conversion and the inconvertible processes for the energy forms. The anti–adjoint property of $N(\varphi)$ indicates that the total of energy conversion caused by the above processes contained in $N(\varphi)$ is zero. The operator $L(\varphi)$ represents the dissipative effects of the system that are the irreversible diabatic processes. Because $(L(\varphi)\varphi, \varphi) \geqslant 0$, the friction and the dissipation are always to lower the quality of energy of the system, i.e., make certain energy become the form of unable work from of able work. The above statement seems to show the negative effect of dissipation. For an open system, however, indeed, the dissipation has particularly positive value[5,6,18,26].

2.2 *Asymptotic Behavior of Solutions —— Existence of Global Attractor (GA)*

Theorem 1 There exists the GA A of OEqs (1) and (2).

The theorem shows that the atmospheric motion governed by OEqs (1) and (2) is closer and closer to A as time increases. The GA A stands for the final state of the system, and is called the atmosphere attractor. It involves the long–time behavior of the system, so it is also termed the climatic attractor. The existence of the GA A reveals the characteristic of the nonlinear adjustment of the atmosphere system to the external forcing, and shows that the property of dissipative structure is a basic characteristic of atmospheric motion. Points out of A represent the transient process, so evidently the system has the irreversible characteristic.

2.3 *Case with Topographic Effect*

The lower boundary conditions of the studies mentioned above are the case of surface viscosity. The topographic forcing, however, is also very important for the atmospheric motion. So a further consideration is whether the above conclusions are true under the topographic effect. The answer is in the affirmation. The references [18,32] have the details of the proof.

2.4 *Case with Non–Stationary External Forcing*

The real external forcing of the atmosphere is non–stationary. For understanding and forecasting of large–scale weather and climate, it is also basic that the regularity of atmospheric motion with the non–stationary external forcing is studied. In reality, the external forcing is always bounded. For the periodic and semi–periodic external forcing, the existence of GA can be proved[18,19] by the theory of non–autonomous differential dynamical system[33–37].

2.5 *Inertial Manifold*

The inertial manifold M_I [36–38] is a limited dimensional invariable smooth manifold. It attracts exponentially the solution trajectory of the system, and the GA is on it. Because the GA may be an unsmooth manifold and the speed of convergence of the solution trajectory to it is not exponential, it is significant that M_I is studied for further dynamical analysis and numerical calculation.

For the equations of large–scale motion[1–3,12, 17], they can be transformed into an initial problem of a class of nonlinear evolution equation[17], and its GA A and inertial manifold M_I can be obtained by the truncated technique[17].

2.6 *Source of Atmospheric Multiple Equilibria (MEs)*

Both the rotating annulus experiments[39–42] and the observational facts[43,44] on the atmospheric circulation show that there can be MEs in the atmospheric circulation under the same external forcing. From the initial studies of Charney *et al.*[45,46] on the atmospheric MEs, many authors[47–52] discussed the MEs phenomenon using the similar method, which is the truncated spectral model. Chou *et al.*[2,18,20] investigated the source of atmospheric MEs.

The problem of MEs is the boundary value problem of solutions for the equations of stationary atmospheric motion. So, Eq.(1) becomes

$$N(\varphi)\varphi + L(\varphi)\varphi = \xi(\varphi), \tag{3}$$

The boundary value conditions are the same as in Eq.(1). For the stationary equations of moist atmospheric motion, it can be proved[18,21] that the stationary solution is either unique or non–existential and in any case multi–solution does not exist if nonlinearity, dissipation or external forcing is left out. This shows that the joint action of nonlinearity, dissipation and external forcing is the source of the atmospheric MEs; i.e., the atmospheric MEs are a kind of the nonlinear mechanism with the interaction between dissipation and external forcing.

2.7 *Effects of External Forcing, Dissipation and Nonlinearity on the Solutions*

It can be proved that the long–time behavior of solutions of the FDN atmospheric system is essentially different from that of the adiabatic non–dissipative (AND) system, the adiabatic dissipative (AD) system, the forced non–dissipative (FND) system or the forced dissipative linear (FDL) system by the Hilbert space methods[5,18,23]: (1) For the AND atmospheric system, there is the conservation of energy and the effect of initial value does not decay up to infinite time, and there is no attractor in it. The existence of attractor is essential distinction between the chaos of the dissipative system and that of the conservative system, although there may also be chaotic motion in the conservative system. Besides, the adiabatic system without dissipation is reversible process. (2) For the AD atmospheric system, it is the global exponential stability, and there is unique final state that is not associated with initial value. The final state is just the stationary solution of the system. From this we can conclude that the regular motion will stop if the system does not obtain the supplementary energy from the outside, and that the motion of dissipative system without external forcing will become more and more identical and monotonous in evolution, and therefore the difference and the particularity in motion will disappear unavoidably. These show that the external forcing is a necessary condition for keeping the activities of the system with dissipation. (3) For the FND atmospheric system, there is no attractors. As time increases, the energy of the system will accumulate continuously and tend to divergence. It follows that the external forcing and the dissipation supplement each other for the real atmosphere, and that the other one must be omitted if one of them is left out, and that it is unmatched in the physical that the other one is left out if one of them is retained. Therefore, they are either left out together for the short–term motion or retained together for the long–term motion. Moreover, the above results show that the dissipation is a global stable factor of the system and a necessary condition for keeping global stability for the system with external forcing. (4) For the FDL atmospheric system, there exists unique stationary solution, and the effect of initial value will decay to zero as time increases, and there exists a unique asymptotic state which is not associated with initial value, i.e., the asymptotic behavior of solutions shows the structure of stationary solution. In addition, there is no chaotic phenomenon in the FDL system. It seems, thus, that the nonlinearity is a necessary condition for arising chaos.

3. APPLICATIONS

3.1 *Three Classes of Time Boundary Layers (TBLs)*

There exists a GA in the atmosphere. Any state out of the GA must be attracted to the GA, and the approximation process to the GA is very quick and the rate of approximation is almost exponential (Out of the inertial manifold it is exponential). Thus, the adjustment to the final state determined by the given external forcing is a fast process, and the state out of the GA is the transient state. The attractor is an invariant set and has the property of relative stability and may be regarded as a kind of equilibrium, so the motion of the state on the attractor is a slowly varying process. It may be concluded that there exist two kinds of characteristic time scales in the atmospheric system, which are the fast adjustment process to the attractor and the slow evolutionary process on the attractor, when the external forcing is stationary or its variation is very slow. Moreover, there exists the third time scale, i.e., the slower evolutionary process of the macroscopic state versus external parameters than the above two kinds of processes, when the external forcing varies. These are three kinds of time scales mainly existing in the atmospheric evolution, which was pointed out by Chou[5,6,11].

Zeng[4,53–57] presented the concept of TBL when he discussed the adjustment process and the evolutionary process in the adiabatic non–frictional atmosphere. Here we extend the concept to the FDN atmosphere; i.e., according to the properties and characteristics of the atmospheric motion we introduce three classes of TBLs, the first, the second and the third TBL. The third TBL is also called the inner TBL. As shown in Fig. 1, the inside of the first TBL is with a quick adjustment process of the state from out of the attractor to the attractor; the outside of it there is an evolution on the attractor, a system corresponding to an FDN system with the stationary external forcing. The second TBL is a layer including the first TBL and the evolutionary process outside of the first. The outside of second TBL is in a slower evolutionary process of macroscopic state against external forcing, i.e. corresponding with the third kind of time scale. In this circumstance, the system should be regarded as an FDN system with the non–stationary external forcing. Additionally, within the time scale of dissipation, the atmosphere may be regarded as an adiabatic and non–frictional system, and there are geostrophic adjustment and evolutionary process as usual. In this case, TBL is the inner (third) TBL of FDN atmosphere, and the corresponding system is the adiabatic non–frictional conservative system. The above discussion reflects the self–similar structure of FDN system in TBL. Based on the concept of three classes of TBL, the adjustment and evolution process of FDN atmosphere can be discussed clearly and be applied to the design of splitting algorithm.

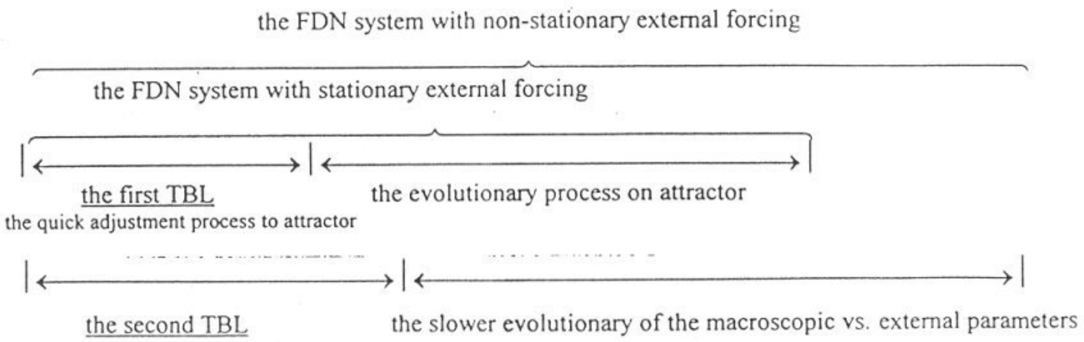

FIGURE 1. The TBL in the FDN system.

3.2 *Constraint Principle of Operator for Simplifying Equations*

The atmosphere equations may be simplified in the investigation of the atmospheric dynamics and designing the numerical models. In order to obtain the simplified equations with the harmonious and consistent dynamical relations that do not distort the essential properties of original equations, the properties of the corresponding operator in the equations before and after simplification should be kept unchanged according to the theoretical results mentioned above. This is a principle of correct simplification. The simplified equations under the principle should not have false source or sink, and should have the GA and keep the global properties and the physical essence of the original equations. Chou[1–3,5] pointed out the principle long ago. The idea is specified in the references [18, 58].

3.3 *Design of Difference Scheme and Computational Stability Analysis (CSA) of FDN Evolutionary Equations*

The numerical solution of differential equations is an approximate method. It involves the construction, the computational stability and accuracy analysis of the numerical scheme[59–62]. The numerical solution of partial differential equations is a method that uses a limited dimensional system instead of an infinite dimensional system, so the operator equation becomes the other state space from one state space. The principle that properties of operators are not altered should be kept in order to hold the physical laws of original system as far as possible. This is a restricted principle of the design of numerical scheme. Only with the principle the better difference schemes can be gotten[18]. It is one of methods keeping the anti–adjoint property of N that the well–known complete square conservation is kept in the design of difference scheme.

Here illustrates briefly an application of the rule in the nonlinear CSA. Hereafter let $N(\varphi^*)$ and $L(\varphi^*)$ be the general discrete forms of $N(\varphi)$ and $L(\varphi)$ respectively, then $N(\varphi^*)$ is an anti–symmetric matrix and $L(\varphi^*)$ a symmetric positive matrix. For Eq.(1), it is very difficult that the CSA for its difference scheme is studied. In general, it is studied only when $\xi \equiv 0$[59–62]. For the case of $\xi \neq 0$, there is no result at present. So, it is necessary that the concept of computational quasi–stability (CQS) is introduced as follows.

Definition 1[17]: If time step length is sufficiently small, the numerical solution computed by the difference method satisfies

$$\|\varphi^{n+1}\| \leqslant \|\varphi^n\| + \tau c, \tag{4}$$

where c is a constant depending on $\|\xi\|$, then the difference scheme is called the computational quasi–stability (CQS) scheme.

When $\xi \equiv 0$, $c=0$. Thus, the CQS in this case is just the CS. The CQS is a necessary condition of CS. In reality, the scheme with CQS is often CS. Here discusses only the basis of CQS for the case of $\xi \neq 0$ because of the complexity of nonlinear problem. The general difference scheme of Eq.(1) is given as follows:

$$(\varphi^{n+1}-\varphi^{n})/\tau+[N(\varphi^{*})+L(\varphi^{*})][a\varphi^{n+1}+(1-a)\varphi^{n}]=\xi, \qquad 0\leqslant a\leqslant 1 \tag{5}$$

Theorem 2 The difference scheme (5) of Eq. (1) is CQS when $1/2\leqslant a\leqslant 1$.

The scheme (5) is CS when $\xi\equiv 0$, which is the same result as in the references [59–62]. Besides, the better results can be obtained for the linear equation of Eq.(1)[18].

3.4 *Constraint Principle of Operator for Splitting Algorithm*

A lot of studies on the splitting algorithm have been carried out[63–73]. The splitting principle is the most important in the splitting algorithm. Depending on the above results, this paper presents the constraint principle of operator of splitting algorithm, i.e., the operators in the splitting equations should be kept the same properties as the corresponding operators in the original equations. It can be radically ensured that the splitting equations under the principle do not distort the intrinsic overall properties of the original equations. Moreover, it is convenient and clear that the equations are split by the principle, and its physical sense is also very distinct.

There is nonlinear adjustment to external forcing in the FDN atmosphere, thus, the first level splitting of Eq.(1) can be written as

$$\frac{\partial\varphi}{\partial t}+L(\varphi)\varphi=\xi, \tag{6}$$

$$\frac{\partial\varphi}{\partial t}+N(\varphi)\varphi=0. \tag{7}$$

That is to say, the processes represented by the external forcing ξ and the dissipation $L(\varphi)$ are the slow evolutionary processes of the system, and the processes represented by $N(\varphi)$ are faster processes than ξ and $L(\varphi)$. Eq.(7) includes fast and slow processes, i.e., the geostrophic adjustment and the evolutionary process. So, we have the second level splitting of (7):

$$\frac{\partial\varphi}{\partial t}+N_i\varphi=0, \qquad i=1,2, \tag{8}$$

where N_1, N_2 represent adjustment and evolutionary process respectively. Both of them are the anti–adjoint operators. In order to improve computational efficiency, the third level splitting of (8) can be carried out. Eq. (6) may also be re–split, i.e.,

$$\frac{\partial\varphi}{\partial t}+L_i\varphi=\xi_i, \tag{9}$$

where all of L_i are self–adjoint operators. The reference [18] gives the concrete examples.

3.5 *Constructing a Few Freedoms to Support the Base of Attractor*

In the infinite dimensional Hilbert space, the limited set of solutions of atmospheric system is able to contract to the limited dimensional manifold, so, theoretically, the asymptotic behavior of the partial differential equations of atmosphere can be described accurately by one limited dimensional ordinary differential equations set. In this way, the problem becomes simple. It is of applied importance. However, it is the crux that the dimension of system is estimated accurately. Unfortunately, there is no a completely and exactly estimated method for obtaining the dimension of attractor so far. Even so, we can still find another way. Because the freedoms to support attractor are infinite, it means that the motion of system will more and more concentrate on a few preferred modes in the evolution process, the successful macro–description for the evolution of system can be gotten if these modes are caught. The reference [26] gave a kind of empirical method for getting a few freedoms to support the base of climatic attractor, i.e., the empirical preferred modes were gotten based on the empirical orthogonal function (EOF) of solutions of the model, and the model is simplified by them. The authors have carried out some numerical experiments using a theoretical model and verified the feasibility and effectiveness of the method. The work on this, however, is rather insufficient.

In addition, this empirical method is based on the real evolution of system (i.e., the observational data), so it can be adapted to the demands of the variable external forcing of the atmosphere. The simplified model got by the method can adjust itself to change in the preferred modes of the system. It can be correspondingly modified as the preferred modes of system change. If we introduce the method to the numerical models, the micro–scale or minor waves can be filtered out and the long–term waves can be retained, so that the numerical forecast can be improved and extended effectively. It follows that the work on such aspect needs to be developed deeply.

3.6 *Essence of Harmonization of Initial Values (IVs) with Models*

It is well–known that the phenomenon of initialization shock will occur if the observational data are used as the IVs of models. This shows that the observational data are generally not harmonious with models, and so it is necessary that the observational IVs are handled. Here it must be pointed out that the errors of IVs are not related to whether the IVs are harmonious with models. Based on the results of this paper, the final state of solutions of all IVs for the dynamical equations of atmosphere is a limited dimensional GA. That is a point set, referred as to A; its space–volume is zero in R^n [3]. It is zero probability that the IVs determined directly by the observational data are on the A. That is to say, the IVs determined directly by the observational data generally are not in the state of attractor, and so they are inconsistent with the real atmosphere and inharmonious with the equations of models. It is simply the essence[11] that the IVs are handled so that they are harmonious with the models in numerical forecast.

3.7 *Numerical Analysis of the Multiple Equilibria*

The MEs of the atmosphere are the result of the joint effects of nonlinearity, dissipation and external forcing. Therefore it is necessary that the FDN models are adopted for the studies of the MEs in the atmosphere. Only in this way can the successful explanations be given for the MEs phenomenon got by the observations. From the initial work of Charney *et al.*[45,46] till the other numerical experiments and analysis for MEs[47–52], all of models are the FDN model. So these studies verified the above conclusions from another aspect.

3.8 *Necessary Conditions of Dynamical Models Describing the Long–Term Process*

According to the qualitative theory of the dynamical equations of the FDN atmosphere, the forcing, the dissipation and the nonlinearity are in general the basic factors that must be considered in the long–term process. Therefore a simplified dynamical model describing the long–term process must be FDN evolutionary equations, neither AND nor linear. Only in this way can the long–time behavior of solutions of the original system not be distorted qualitatively; otherwise we cannot obtain the convincing long–term numerical weather forecast and numerical climatic forecast.

3.9 *Importance of the Information on the Natural Patterns of Anomalous Underlying Surface Factors*

Based on the result of the FDN system with the nonlinear adjustment to the external forcing, the forcing effects of underlying surface factors, specially the information on the natural patterns of significant anomalous underlying surface factors and their nonlinear interactions, should be firstly considered for the numerical simulation and forecast of the monthly and seasonl time scale. The references [29,30] made the significant studies for this, and applied the present approach to the numerical models for improving the forecast for the precipitation of the flood season in China. Their results indicate that the approach is great promise for improving the long–term prediction, so it is worthy of further work.

3.10 *Prediction for One Class of the Meso–Scale System*

The physical models of a great class of meso–scale severe convective system, theoretically, can be dealt with an FDN system subject to the force of underlying surface and latent heat release under the control of large–scale background field with the given external fo rcing. The observational facts can also display this point[27,74,75]. Based on the conclusion, the references [27,28] discussed the characteristics and the predictive methods of this class of meso–scale convective system caused by the underlying surface force and pointed out that the meso–scale force of underlying surface plays a decisive role in the formation and development of this kind of meso–scale system. However, the problems on the characteristics and numerical forecast of this class of meso–scale convection system determined by the large–scale background field and underlying surface force await further investigation.

4. SUMMARY, PROBLEM AND VISTA

The long–term numerical weather forecast and the numerical climatic forecast involve the long–time evolution processes of the system. Their theoretical foundations are the FDN dynamics. Based on the complete dynamical equations of the moist atmosphere, this paper discusses systematically the global asymptotic behavior of solutions of FDN dynamical equations of the atmosphere, sums up and penetrates into the known work on this subject, presents some new viewpoints and applications, and tries to throw out a minnow to catch a whale for developing further work. Obviously, the studies of the FDN dynamics are still just begun. Moreover, there are many existed problems and further theoretical and applied subjects.

According to the results of the qualitative theory, there exists the GA in the atmosphere, and it is also divided up by some attractors and the other unimportant and non–attractor invariable point sets. It is still not given by the qualitative theory at present that what are the structures and the properties of these attractors under the specific external forcing. The problems about these aspects are still far from being settled because of their difficulties. However, the cell–mapping method is one of effective means for analyzing them[5,6,9,31,76].

There exist the chaotic phenomena in time as well in space, i.e. the space–time chaos in the atmosphere. Chaos produces in the certain space domains, but does not occur in the other space regions. The finite dimensional dynamical system has not this property. The great majority of the studies in the past focus on the chaos in the time domain, little on the chaos in the space domain. So the theory on space chaos is the current pressing subject. It is a forward position which may well make significant break–through that the theory on the space chaos of atmosphere system is developed and the space inhomogeneity of bifurcation and the space chaos in atmosphere system are revealed by the observational data and the theory[8].

The qualitative theory, on the one hand, is to explain the observational phenomena in the atmosphere; on the other hand, to apply to numerical models in order to improve numerical forecast. The researches about these aspects have been carried out, and they also appear quite promising, but it is still miles apart between them and operational use. Hence we should devote major efforts to developing them in the future.

The qualitative theory has also the important applications to the reliability analysis of numerical calculations. The reliability of numerical methods for nonlinear differential equations with the transient chaos or the chaos is of sufficient practical importance to merit research at present, and it is very important that the differences between the physical chaos and the computational chaos are made clear in real application. The subject is of great value in the computational meteorology as well as in the whole computational mathematics. It involves the change of idea from the idealization of infinite precision to the reality of finite precision, to which is paid little attention now. In one word, the qualitative theory is closely associated with the numerical calculation. The results of qualitative theory not only are the important theoretical foundation of numerical calculations, but also are helpful to improve numerical methods. The results of numerical methods, however, provide concrete and abundant materials to the qualitative theory. The combination of qualitative analysis and numerical calculations contributes to reveal the evolutionary law of FDN dynamical system. As Chou[5,6] pointed out, it will be one of important characteristics of atmospheric sciences in the 21th century that the qualitative analysis and the numerical calculation supplement each other.

ACKNOWLEDGEMENT

This work was supported by the State Key Research Project on Dynamics and Prediction Theory of the Climate.

REFERENCES

[1] Chou Jifan, 1983: Some properties of operators and the decay of effect of initial condition *Acta Meteor. Sinica*, **41**(4), 385–392. (in Chinese)

[2] Chou Jifan, 1986: *Long–Term Numerical Weather Prediction*, Beijing, China Meteorological Press. (in Chinese)

[3] Chou Jifan, 1990: *New Advances in Atmospheric Dynamics*, Lanzhou, Lanzhou University Press. (in Chinese)

[4] Zeng Qingcun, 1979: *Mathematical and Physical Foundations of Numerical Weather Prediction*, Beijing, Science Press. (in Chinese)

[5] Chou Jifan, 1995: Global analysis of the dynamical equations of atmospheric motion, *J. Beijing Meteor. College*, No.1, 1–12. (in Chinese)

[6] Chou Jifan, 1995: Some advances and trends in the atmospheric dynamics, *The Forward Position and Prospects for Modern Atmospheric Sciences*, Beijing, China Meteorological Press, 71–75. (in Chinese)

[7] Chou Jifan, Liu Shida, Liu Shikuo, 1994: *Nonlinear Dynamics*, Beijing, China Meteorological Press. (in Chinese)

[8] Chou Jifan, 1997: Advances, problems and prospects of nonlinearity and complexity research in the atmospheric sciences, *Bulletin of Chinese Academy of Sciences*, **12**(5), 325–329. (in Chinese)

[9] Chou Jifan, 1986: Some general properties of the atmospheric model in H space, R space, point mapping, cell mapping, *Proceedings of International Summer Colloquium on Nonlinear Dynamics of the Atmosphere*, 10–20 Aug., 1986, Beijing: Science Press, 187–189.

[10] Guo Bingrong, Du Xingyuan, Chou Jifan, 1986: *Applications of Mathematical Methods to Atmospheric Sciences*, Beijing, China Meteorological Press. (in Chinese)

[11] Chou Jifan, 1995: Theory and new methods on four-dimensional data assimilation, *Some New Techniques in Numerical Weather Prediction*, Beijing, China Meteorological Press, 262–294. (in Chinese)

[12] Chou Jifan, 1995: Discussions on comprehensive and consensus short-term climatic forecast, *Xinjiang Meteor.*, **18**(5), 1–7. (in Chinese)

[13] Wang Shouhong, Huang Jianping, Chou Jifan, 1990: Some properties of solutions for the equations of large-scale atmosphere, nonlinear adjustment to the time-independent external forcing, *Science in China (Ser B)*, **33**(4), 476.

[14] Lions, J. L., Temam R., Wang S., 1992: New formulations of the primitive equations of atmosphere and applications, *Nonlinearity*, **5**, 237–238.

[15] Wang, S., 1992: Attractors for the 3D baroclinic quasi-geostrophic equations of large scale atmosphere, *J. Math. Anal. Appl.*, **165**, 266–283.

[16] Lions, J. L., Temam R., Wang S., 1993: Mathematical models and mathematical analysis of the Ocean / Atmosphere system, *C. R. Acad. Sci. Paris*, Ser. I, **316**, 211–215.

[17] Lions, J. L., Temam R. and Wang S., 1992: On the equations of large-scale ocean, *Nonlinearity*, **5**, 1007–1053.

[18] Li Jianping, 1997: Qualitative Theory of the Dynamical Equations of Atmospheric and Oceanic Motion and Its Applications, Lanzhou University, Ph.D Dissertation, pp209. (in Chinese)

[19] Li Jianping, Chou Jifan, 1997: Existence of atmosphere attractor, *Science in China (Ser D)*, **40**(2), 215–224.

[20] Li Jianping, Chou Jifan, 1996: The property of solutions for the equations of large-scale atmosphere with the non-stationary external forcing, *Chinese Science Bulletin*, **41**(7), 587–590.

[21] Li Jianping, Chou Jifan, 1996: Source of atmospheric multiple equilibria, *Chinese Science Bulletin*, **41**(24), 2074–2077.

[22] Li Jianping, Chou Jifan, 1998: The asymptotic behavior of solutions for the equations of the moist atmosphere, *Acta Meteor. Sinica*, **56**(2): 187–198, (in Chinese)

[23] Li Jianping and Chou Jifan, 1997: The effects of external forcing, dissipation and nonlinearity on the solutions of atmospheric equations, *Acta Meteor. Sin.*, **11**(1), 57–65.

[24] Li Jianping and Chou Jifan, 1997: Further study on the properties of operators of atmospheric equations and the existence of attractor, *Acta Meteor. Sin.*, **11**(2), 216–223.

[25] Li Jianping, Chou Jifan, 1996: Some problems existed in estimating fractal dimension of attractor with one-dimensional time series, *Acta Meteor. Sinica*, **54**(3), 312–329. (in Chinese)

[26] Zhang Banglin, Chou Jifan, 1992: Applications of empirical orthogonal function (EOF) to numerical climatic simulation, *Science in China (Ser B)*, **35**(1), 92–101.

[27] Li Zhijin, Chou Jifan, 1993: A class of severe convective systems influenced by underlying surface force and its prediction, *Science in China (Ser D)*, **23**(10), 1114–1120. (in Chinese)

[28] Li Zhijin, 1992: Characteristics of A Class of Severe Convection Systems Caused by Underlying Surface Force and Its Predictive Methods, Lanzhou University, Ph.D. Dissertation, pp62.

[29] Dong Wenjie, Chou Jifan, 1996: Preliminary studies on comprehensive consensus forecast for precipitation of flood season improved by numerical model, *Climatic Forecast Studies*, Beijing, China Meterological Press, 119–130. (in Chinese)

[30] Dong Wenjie, 1996: Statistic Analysis, Model Studies and Predictive Methods for Anomalous Precipitation during Summer in China, Lanzhou University, Ph.D. Dissertation, pp129. (in Chinese)

[31] Guo Bingrong, Jiang Jianmin, Fan Xingang, Zhang Hongliang, Chou Jifan, 1996: *Nonlinear Characteristics and Predictive Theories on Climatic System*, Beijing, China Meteorological Press. (in Chinese)

[32] Li Jianping, Chou Jifan, 1998: The long time behavior of solutions of atmospheric equations with topographic effects, *J. Beijing Meteor. College*, (1): 1–12, (in Chinese)

[33] Haraus, A., 1988: Attractors of asymptotically compact processes and application to nonlinear partial differential equations, *Comm. P. D. E.*, **13**, 1383–1414.

[34] Chepyzhov, V. V., Vishik M. I., 1994: Attractors of non-autonomous dynamical systems and their dimension, *J. Math. Pures. Appl.*, **73**, 279–333.

[35] Chepyzhov, V. V., Vishik M. I., 1993: Non-autonomous dynamical systems and their dimension, *Russ. J. Math. Physic.*,**1**(2), 165–190.

[36] Guo Buolin, 1995: *Nonlinear Evolutionary Equations*, Shanghai, Shanghai Scientific and Technological Press, 183–343. (in Chinese)

[37] Yin Chaoyang, 1996: The Long Time Behavior of Non-Autonomous Infinite Dynamical System, Lanzhou University, Ph.D. Dissertation, pp120.

[38] Foias, C., Sell G. R. and Temam R., 1988: Inertial manifolds for nonlinear evolutionary equations, *J. Diff. Eqa.*, **73**, 309–353.

[39] Hide R., 1953: Some experiments on thermal convection in a rotating liquid, *Quart. J. R. Meteor. Soc.*, **79**, 161.

[40] Hide, R., 1953: Fliuid motion in the earth's core and some experiments on thermal convection in a rotating liquid, Fluid Models in Geophysics, Proc. 1st Sympos on the Use of Models in Geophys Fluid Dynamics, Baltimore, 101–106.

[41] Fultz, D., 1953: A study of certain thermally and mechanically driven fluid systems of meteorogical interest, Proc. 1st Sympos on the Use of Models in Geophys Fluid Dynamics, Baltimore, 27–63.

[42] Hide, R., 1958: An experimental study of thermal convection in a rotating liquid, *Phil. Trans. Roy. Soc. London*, (A), **250**, 441–478.

[43] Ye Duzheng, Tao Shiyan, Li Maicun, 1958: The abrupt change of atmospheric circulation over northern hemisphere during June and October, *Acta Meteor. Sincia*, **29**(3), 249–263. (in Chinese)

[44] Ye Duzheng, Gao Yuxi, 1979: *Qinghai–Xizhang Plateau Meteorology*, Beijing, Science Press, 278. (in Chinese)

[45] Charney, J. G., de Vore J. G., 1979: Multiple flow equilibria in the atmosphere and blocking, *J. Atmos. Sci.*, **36**, 1205–1216.

[46] Charney, J. G. and Straus D. M., 1980: Form-drag instability, multiple equilibria and propagating planetary waves in baroclinic, orographically forced, planetary waves system, *J. Atmos. Sci.*, **37**, 1157–1176.

[47] Li Maicun, Luo Zhexian, 1986: Effects of moist processes on mutiple equilibria and subtropical flow patterns, *Science in China (Ser B)*, (1), 106–112. (in Chinese)

[48] Li Maicun, Luo Zhexian, 1986: Bifurcation from equilibrium state to periodic state and two kind of low-frequency fluctuation of subtropical flow patterns, *Science in China (Ser B)*, (5), 551–560. (in Chinese)

[49] Jin Feifei, Zhu Baozhen, 1986: Interaction between forced wave, free wave and meridional flow I. Bifurcation of equilibrium state, *Science in China (Ser B)*, (6), 663–672. (in Chinese)

[50] Li Maicun, Luo Zhexian, 1986: Nonlinear mechanism of abrupt change of atmospheric circulation during June and October, *Science in China (Ser B)*, (1), 106–112. (in Chinese)

[51] Miao Jinhai, Ding Minfang, 1985: Abrupt change of atmospheric multiple equilibria and jump of subtropical high pressure under thermal forcing, *Science in China (Ser B)*, (1), 87–96. (in Chinese)

[52] Dong Buwen, Chou Jifan, 1988: Observational analysis and theoretical simulation for seasonal variation of the ridge line of subtropical high pressure over the west Pacific, *Acta Meteor. Sinica*, **46**(3), 361. (in Chinese)

[53] Zeng Qingcun, 1963: Adjustment process and eveloution process of atmosphere, I., *Acta Meteor. Sinica*, **33**(2), 163–174. (in Chinese)

[54] Zeng Qingcun, 1963: Adjustment process and eveloution process of atmosphere, II., *Acta Meteor. Sinica*, **33**(3), 281–289. (in Chinese)

[55] Zeng Qingcun, Ye Duzheng, 1981: The advance in investigation of the problems of the adaptation processes in the rotating atmosphere, I, *Scientia Atmospherica Sincia*, **4**(4), 379–393. (in Chinese)

[56] Zeng Qingcun, ye Duzheng, 1982: The advance in investigation of the problems of the adaptation processes in the rotating atmosphere, II, *Scientia Atmospherica. Sincia*, **5**(1), 101–112. (in Chinese)

[57] Zeng Qingcun, 1979: Nonlinear interaction and rotational adjustment process in rotational atmosphere, *Scinece in China (Ser B)*, (10), 986–995. (in Chinese)

[58] Luo Zhexian, 1986: Pattern of large-scale motion and dissipative structure, *Acta Meteor. Sinica*, **44**(2), 140–148. (in Chinese)

[59] Zeng Qingcun, 1978: Some problems on computational stability, *Scientia Atmospherica Sinica*,

2(3), 181–191. (in Chinese)

[60] Zeng Qingcun, Ji Zhongzhen, 1981: On the computational stability of evolution equations, *Mathematica Numerica Sinica*, **3**(1), 79–86. (in Chinese)

[61] Ji Zhongzhen, Zeng Qingcun, 1982: Construction and apllication of difference scheme of evolution equations, *Scientia Atmospherica Sinica*, **6**(1), 88–94. (in Chinese)

[62] Ji Zhongzhen, Wang Bin, Zeng Qingcun, 1995: Complete energy conservation difference methods and its application, *Some New Techniques in Numerical Weather Prediction*, Beijing, China Meteorological Press, 26–46. (in Chinese)

[63] Zeng Qingcun, Yuan Chongguang, 1980: The Splitting algorithm for solving atmospheric equtions, *Chinese Science Bulletin*, **25**(18), 842–845. (in Chinese)

[64] Zeng Qingcun, Zhang Xuehong, 1982: Perfectly energy conservation time–space finite–difference schemes and the consistent split method to solve the dynamical equations of compressible fluid, *Scinece in China (Ser B)*, **25**(8), 866–880.

[65]]Marchuk, G. I., 1968: Short term weather prediction by splitting of the complete hydrodynamic equations, *Proc. WMD/ IUGG Symp. Num, Wea. Prod.*, Tokyo, Nov 26–Dec **4**, II, 1–8.

[66] Marchuk, G. I., 1972: *Numerical Methods in Weather Prediction*, New York: Academic Press.

[67] Burridge, D. M., 1975: A split semi–implicit reformulation of the Bushby–Timpson 10–level model, *Quart. J. R. Meteor. Soc.*, **101**, 777–792.

[68] Mesinger, F., Arakawa A., 1976, Numerical methods used in atmospheric models, I, GARP Publications Series 12, Geneve.4.

[69] Gadd, A. J., 1978: A split explicit intergration scheme for NWP, *Quart. J. R. Meteor. Soc.*, **104**, 569–582.

[70] Chen Qiushi, 1980: A split explicit calculation method for analyzing the physical process of synoptic situation variation, Collected Papers of the Chinese Second Numerical Weather Prediction Colloquium, Beijing, Science Press, 271–282. (in Chinese)

[71] Ji Zhongzhen, Wang Bin, 1991: Further discussion on construction and application of difference scheme of evolution equations, *Scientia Atmospherica Sinica*, **15**(2), 72–81. (in Chinese)

[72] Wang Bin, Ji Zhongzhen, Zeng Qingcun, 1995: Preliminary discussion on the theory of split algorithm, *Matmethica Numerica Sinica*, **17**(2), 115–126. (in Chinese)

[73] Yao Jianguo, Liao Dongxian, 1993: Two problems on the split algorithm, *Scientia Atmospherica Sinica*, **17**(4), 434–441. (in Chinese)

[74] Lei Yuxun, 1980: Some problems on the severe convective weather, *Scientia Atmospherica Sinica*, **4**(1), 94–102. (in Chinese)

[75] Ye Duzheng, Li Maicun, 1980: Multi–scale characteristics of the every variety of atmospheric motion, Collected Paper of The Chinese Second Numerical Weather Prediction Colloquium, Beijing, Science Press, 81–192. (in Chinese)

[76] Hsu, C. S., 1987: *Cell–to–Cell Mapping — A Method of Global Analysis for Non–linear System*, Springer–Verlag.

[77] Hao Bailin, 1983: Bifurcation, chaos, strange attractor, turbulence and other inherent randomness on deterministic systems, *Advances in Physics*, **3**(3), 329–415. (in Chinese)

[78] Babin, A. V. and Vishik M. I., 1983: Attractors of partial differential equations and estimate of their dimension, *Uspekhi. Mat. Nauk*, **38**, 133–187.

[79] Teman, R., 1988: *Infinite Dimensional Dynamical System in Mechanics and Physics, Applied Math Sci Series*, Vol. 38, New York: Springer–Verlag.

[80] Ladyzhenskaya O. A., 1983: Attractor for the Navier–Atokes system and for parabolic equations and estimates of their dimension, *J. Soviet. Math.*, **28**, 619–627.

[81] Constantin P., Foias C., Temam R., 1985: Attractors representing turbulent flows, *Memoirs Amer. Math. Soc.*, **314**.